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Regression Analysis Applied to Shotgun Range-of-Fire Estimations: Results of a Blind Study

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ABSTRACT: Blind studies were conducted to determine if data from one lot of shotgun ammunition could be used to estimate the range of fire of a pattern fired with another lot of ammunition. Thirty pellet patterns were test fired with 00 buckshot and No. 9 birdshot at ranges of 3.1 to 15.2 m (10 to 50 ft) using a 12-gage shotgun. Regression analyses were performed on the spreads of the pellet patterns (calculated as the square root of the area of the smallest rectangle that would enclose the pellet pattern). In both cases linear functions best described the relationship between the range of fire and the spread of a pellet pattern. For the blind study, ten pellet patterns were fired at randomly selected ranges using a different lot of No. 9 birdshot cartridges from that used to determine the regression equation. In the case of the 00 buckshot ammunition, ten pellet patterns were fired at randomly selected ranges using 00 buckshot cartridges from a lot different from that used to determine the regression equation; ten pellet patterns were also fired at randomly selected ranges using 00 buckshot cartridges produced by a different company. Six pellet patterns were fired at a range of 15.2 m (50 ft) with ammunition from each lot used to fire the questioned pellet patterns. The spreads of these pellet patterns were statistically different (at the 95% level) from those fired at the same range with the ammunition used to obtain the regression equations. The means of the spreads of these six pellet patterns were used to calculate scaling factors for the questioned pellet patterns. The scaled spreads of the questioned pellet patterns were inserted into the appropriate regression equation to obtain the estimated range of fire for each of the questioned patterns. The 99% confidence intervals for the estimated ranges of fire were also calculated using the results of the regression analyses. In all cases the actual range of fire for each questioned pellet pattern fell within the 99% confidence interval for the estimated range of fire.

KEYWORDS: criminalistics, ballistics, shotguns

Frequently, firearms examiners are asked to estimate the range from which a shotgun pellet pattern was fired. Previous papers [*1-3*] have explored the use of regression analysis in estimating what the range of fire for a shotgun pellet pattern was. Rowe and Hanson [*3*] have

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described the results of a blind study in which regression analysis was used to estimate the actual ranges of fire for ten shotgun pellet patterns fired at ranges between 3.9 and 15.2 m (10 and 50 ft). In their study, two 12-gage shotguns, each firing a different type of ammunition, were used. The test-fired patterns on whose spreads regression analysis was performed were fired with the same lots of ammunition as those used to fire the questioned patterns.

We now describe the results of a blind study in which regression analysis was used to estimate the ranges of fire for shotgun pellet patterns fired with the same shotgun using different lots of similar ammunition. The questioned pellet patterns were scaled to make them comparable to those of the test-fired patterns; the scaled spreads of the questioned patterns were then used to obtain range-of-fire estimates and associated confidence intervals.

Experimental Procedure

A Stevens Model 77E 12-gage shotgun with a 50-cm (20-in.) full choke barrel was used to fire all the pellet patterns used in this study. Thirty pellet patterns were fired with Federal 00 Magnum buckshot cartridges (70 mm [2¾ in.], twelve pellets per load, Lot No. 2127). Six patterns were fired at each of the following distances: 3.1, 6.1, 9.1, 12.2, and 15.2 m (10, 20, 30, 40, and 50 ft). Ten pellet patterns were fired with both Federal 00 Magnum buckshot cartridges (Lot No. 2120) and Winchester 00 Super-X Magnum buckshot cartridges (70 mm [2¾ in.], twelve pellets per load, Lot. No. F101UA41) at ranges selected using a table of random pairs of digits [*4*]. These patterns served as the questioned patterns for the blind study. An additional six shots were fired at a range of 15.2 m (50 ft) with ammunition from both of these lots. These latter patterns were used to obtain scaling factors for the spreads of the pellet patterns fired with Lot Nos. 2120 and F101UA41. As explained below, the scaling factors were subsequently used in estimating the ranges of fire for the questioned patterns.

Thirty pellet patterns were fired with Peters Heavy Target Load No. 9 birdshot cartridges (70 mm [2¾ in.], 32-g [1⅛-oz] load, Lot No. LBU 06H506) at the same distances used for the 00 buckshot cartridges. Ten pellet patterns were fired at randomly determined distances [*4*] using Lot No. LAW 081506 of the same type of Peters ammunition. Six pellet patterns were fired at a range of 15.2 m (50 ft) with ammunition from Lot No. LAW 081506 to obtain a scaling factor for the spreads of the questioned pellet patterns fired with this lot of ammunition.

All firing was done on an indoor firing range. The targets were large sheets of paper mounted on a wooden backstop. All ranges of fire were measured from the muzzle of the shotgun to the target. The spreads of the pellet patterns were measured by calculating the square root of the area ($\sqrt{A}$) of the smallest rectangle that completely enclosed the pellet patterns. This method was chosen because of the findings of Wray et al. [*2*], who showed that this measure of the size of a pellet pattern is a linear function of the range of fire. Wray et al. also found that the confidence intervals for estimated ranges of fire obtained with this measure of pellet pattern spread were not significantly different from those obtained by other, more complicated methods.

The composition of each type of shotshell used in this study was examined (Figs. 1-3). Some differences in construction between the Federal and Winchester cartridges were noted: A white granulated plastic material was packed around the shot in the Winchester cartridges (Fig. 2), while the space between the shot in the Federal cartridges (Fig. 1) was void. The Winchester cartridges were also sealed with a crimp, while the Federal cartridges were sealed with a disk.

The 00 buckshot and No. 9 birdshot patterns were fired and their spreads measured by three of the authors (C. H. F., W. A. R., and R. H. W.). The remaining author performed the regression analyses and calculated the estimated ranges of fire for the questioned patterns. He did not know the actual ranges of fire of the questioned patterns.

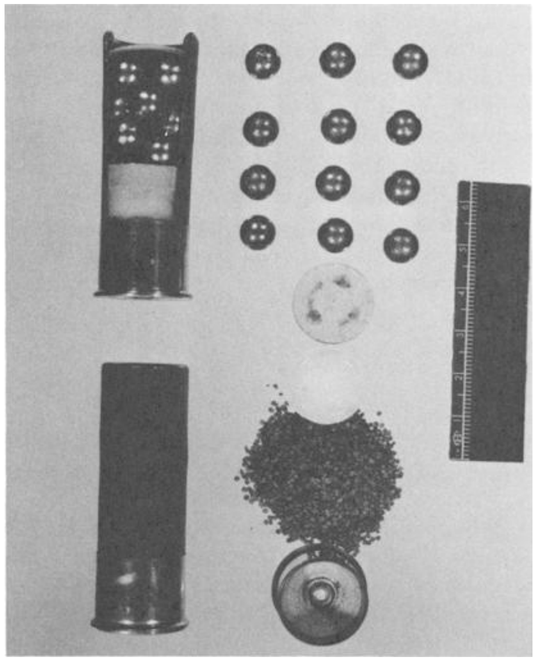

FIG. 1—*Components of the Federal 00 buckshot cartridge.*

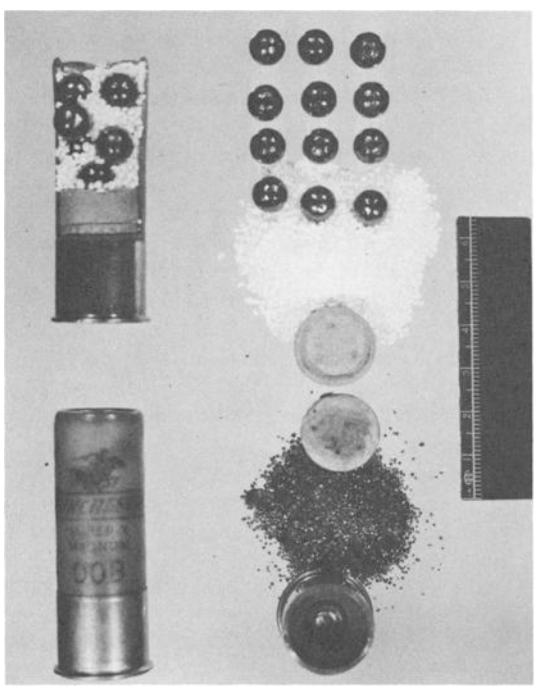

FIG. 2—*Components of the Winchester 00 buckshot cartridge.*

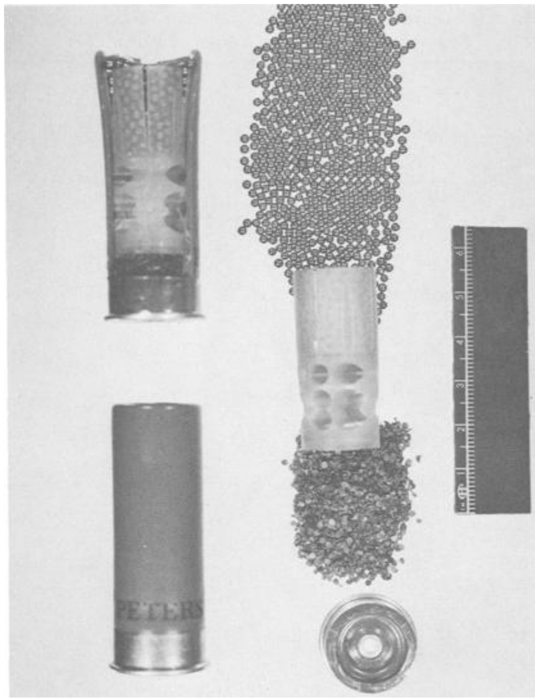

FIG. 3—*Components of the Peters No. 9 shot shotgun cartridge.*

Regression Analysis

The regression analyses were performed using standard methods that have been discussed in detail elsewhere [*2*]. Regressions were performed on the standard deviations of $\sqrt{A}$ as a function of the range of fire to obtain an interpolation function for the estimation of the standard deviations of $\sqrt{A}$ at the estimated ranges of fire of the questioned pellet patterns. The estimated standard deviations are required for the calculation of the confidence intervals of the estimated ranges of fire. Because of the variation of the standard deviations of $\sqrt{A}$ with range of fire, weighted regressions were performed on the values of $\sqrt{A}$ as a function of the range of fire. The weight chosen was proportional to 1/(estimated standard deviation)^2. Various regression models were tested, including linear, quadratic, and cubic functions of the range of fire. For both sets of test-fired patterns, linear functions were found to represent adequately the variation of the standard deviation of $\sqrt{A}$ and the variation of $\sqrt{A}$ itself with the range of fire, as indicated in Tables 1 through 4 and Figs. 4 through 7.

To apply an F test to the result of the regressions on the $\sqrt{A}$ and to use the t statistic to calculate the confidence intervals for the estimated ranges of fire, the weighted residuals of $\sqrt{A}$ must be shown to have been drawn from a normally distributed population having zero mean [*5*]. The weighted residual, in this case, is

$$\frac{\text{(Observed spread} - \text{Estimated spread)}}{\text{Estimated standard deviation}} \qquad (1)$$

TABLE 1—*Spread of shotgun pellet patterns versus range of fire for Federal 00 buckshot cartridges, Lot No. 2127.*

Range of Fire, ft	$\sqrt{A}$, cm	Mean, cm	Standard Deviation, cm	Estimated $\sqrt{A}$, cm	Estimated Standard Deviation, cm	Weighted Residuals
10	5.34	5.48	0.46	5.37	0.67	−0.05
	4.96					−0.62
	5.44					+0.11
	5.45					+0.12
	6.34					+1.46
	5.34					−0.05
20	11.90	10.27	2.07	10.98	1.74	+0.53
	7.21					−2.16
	13.06					+1.19
	9.05					−1.11
	10.49					−0.28
	9.90					−0.62
30	17.63	15.39	3.18	16.59	2.82	+0.53
	13.98					−0.93
	12.85					−1.33
	18.80					+0.78
	17.99					+0.50
	11.05					−1.96
40	23.10	22.81	3.02	22.21	3.90	+0.23
	21.35					−0.22
	28.47					+1.61
	23.09					+0.23
	20.27					−0.50
	20.60					−0.41
50	30.47	30.41	5.37	27.82	4.98	+0.53
	31.67					+0.77
	31.76					+0.79
	39.08					+2.26
	25.62					−0.44
	23.86					−0.80

[a]1 ft = 0.3048 m.

The weighted residuals for both sets of test-fired pellet patterns are given in the last columns of Tables 1 and 2. So-called "normal plots" are commonly used in regression analysis to examine the distribution of residuals. The residuals are ranked in ascending or descending order and then the fraction of the total residual population represented by each residual is plotted on a probability ordinate versus the magnitude of the residual. If the residuals are drawn from a normally distributed population, the resulting graph should be a straight line. The abscissa of the 50% point on the graph is the mean of the distribution. As can be seen in Figs. 8 and 9, normal plots of the weighted residuals are consistent with the hypothesis that they were drawn from a normal population with zero mean. Similar plots of weighted residuals for higher-order regression models revealed significant deviations from normal distributions; therefore, these models were rejected.

Analysis of variance (ANOVA) for the regressions on the standard deviations of $\sqrt{A}$ and on $\sqrt{A}$ for linear functions of the range are given in Tables 5 through 8. The regressions on the standard deviations were significant at the 95% confidence level, while the weighted regressions on $\sqrt{A}$ were significant at the 99% confidence level.

TABLE 2—*Spread of shotgun pellet patterns versus range of fire for Peters No. 9 shot cartridges, Lot No. LBU 06H506.*

Range of Fire, ft	$\sqrt{A}$, cm	Mean, cm	Standard Deviation, cm	Estimated $\sqrt{A}$, cm	Estimated Standard Deviation, cm	Weighted Residuals
10	8.12	6.77	0.71	6.69	1.10	+1.30
	6.53					−0.15
	6.89					+0.18
	6.58					−0.10
	6.48					−0.19
	6.05					−0.58
20	16.80	15.67	2.12	15.36	1.69	+0.85
	13.10					−1.34
	16.44					+0.64
	15.63					+0.16
	18.67					+1.96
	13.39					−1.17
30	22.88	22.52	2.40	24.03	2.28	−0.50
	21.59					−1.07
	18.89					−2.25
	21.68					−1.03
	25.78					+0.77
	24.34					+0.14
40	35.56	32.47	2.82	32.70	2.86	+1.00
	33.78					+0.38
	35.07					+0.83
	29.94					−0.97
	28.56					−1.45
	31.95					−0.26
50	37.50	43.02	3.30	41.37	3.45	−1.12
	47.23					−1.70
	41.62					+0.07
	43.88					+0.73
	44.93					+1.03
	43.00					+0.47

[a]1 ft = 0.3048 m.

TABLE 3—*Results of regression analysis (Federal 00 buckshot cartridges, Lot No. 2127).*

A. Regression on standard deviation versus range

$$\text{standard deviation} = -0.41 + 0.11\,X$$
$$R = 0.952$$

B. Regression on spread of shotgun pellet patterns

$$\sqrt{A} = -0.24 + 0.56\,X$$
$$R = 0.959$$

X = range of fire, ft.
R = correlation coefficient.
$\sqrt{A}$ = square root of area, cm.

TABLE 4—*Results of regression analysis (Peters No. 9 shot cartridges, Lot No. LBU 06H506).*

A. Regression on standard deviation versus range

$$\text{standard deviation} = 0.05 + 0.058\ X$$
$$R = 0.948$$

B. Regression on spread of shotgun pellet patterns

$$\sqrt{A} = -1.98 + 0.86\ X$$
$$R = 0.985$$

X = range of fire, ft.
R = correlation coefficient.
$\sqrt{A}$ = square root of area, cm.

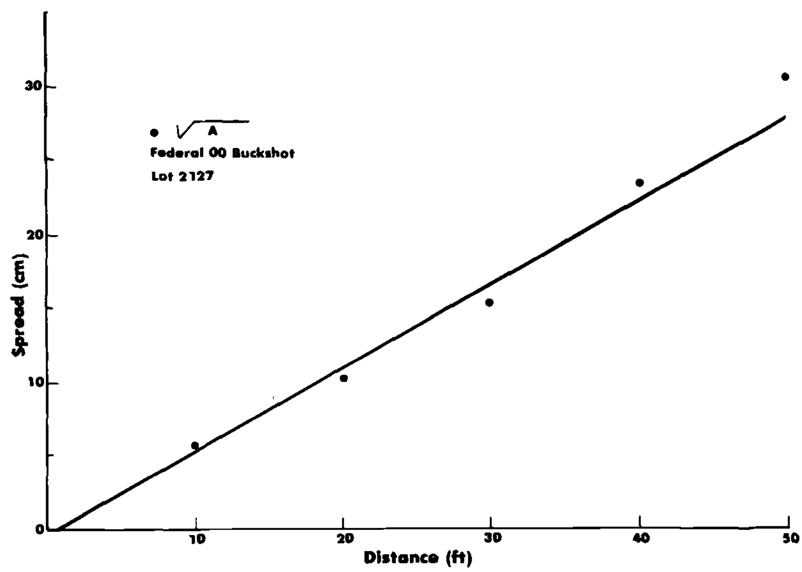

FIG. 4—$\sqrt{A}$ *versus range of fire for Federal 00 buckshot, Lot No. 2127. Only mean values are plotted.*

Estimation of Ranges of Fire

Comparisons of the spreads of the pellet patterns produced by 00 buckshot Lots No. 2120 and F101UA41 at 15.2 m (50 ft) with patterns fired at the same range with Lot No. 2127 were made using the z statistic [*6*] for unpaired replicate samples. If one set of n_1 items of data has mean $\bar{x}_1$ and standard deviation s_1, while a second set of n_2 items has mean $\bar{x}_2$ and standard deviation s_2, the hypothesis that $\bar{x}_1 - \bar{x}_2 = d$ may be tested by calculating

$$z = (\bar{x}_1 - \bar{x}_2 - d)/(s_1^2/n_1 + s_2^2/n_2)^{1/2} \tag{2}$$

If the magnitude of the calculated z-value exceeds the tabulated z-value for a given confidence level, then the hypothesis may be rejected at that confidence level. The z statistic calculated for $d = 0$ showed that the spreads of the pellet patterns fired at a range of 15.2 m

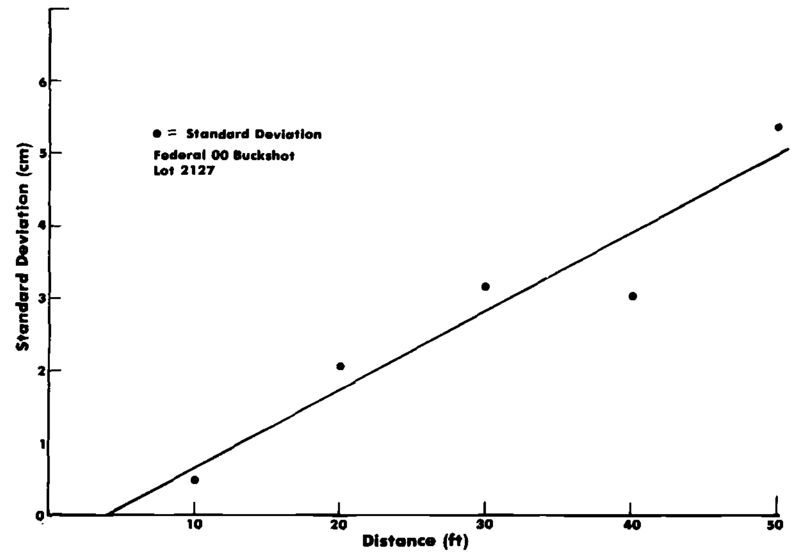

FIG. 5—*Standard deviation versus range of fire for Federal 00 buckshot, Lot No. 2127. Best-fit regression line is shown.*

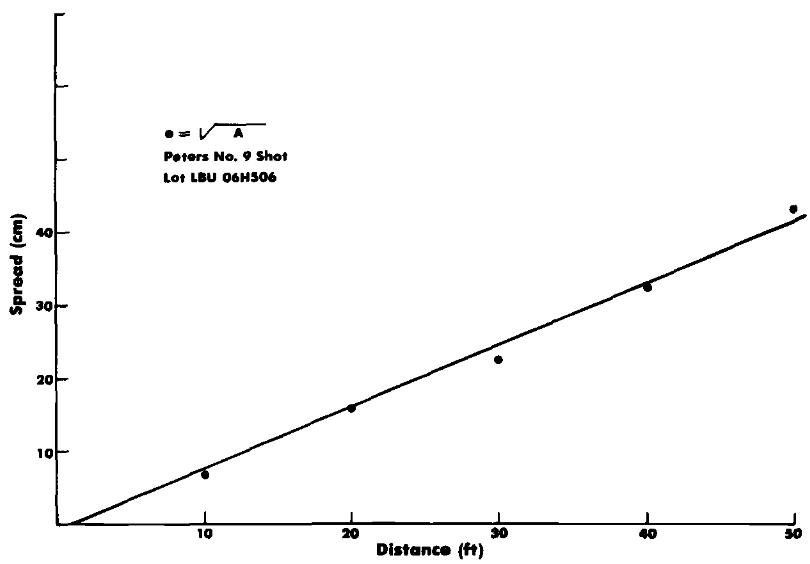

FIG. 6—$\sqrt{A}$ *versus range of fire for Peters No. 9 shot, Lot No. LBU 06H506. Only mean values are plotted.*

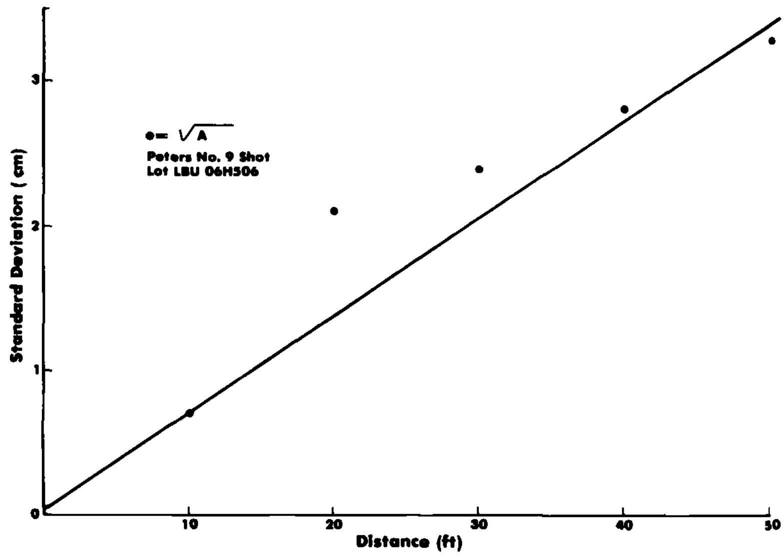

FIG. 7—*Standard deviation versus range of fire for Peters No. 9 shot, Lot No. LBU 06H506. Best-fit regression line is shown.*

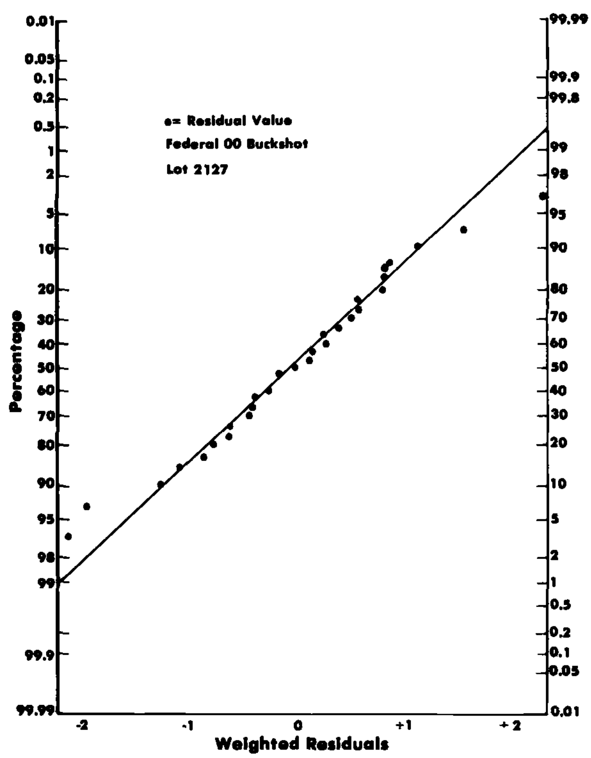

FIG. 8—*Normal plot for Federal 00 buckshot, Lot No. 2127.*

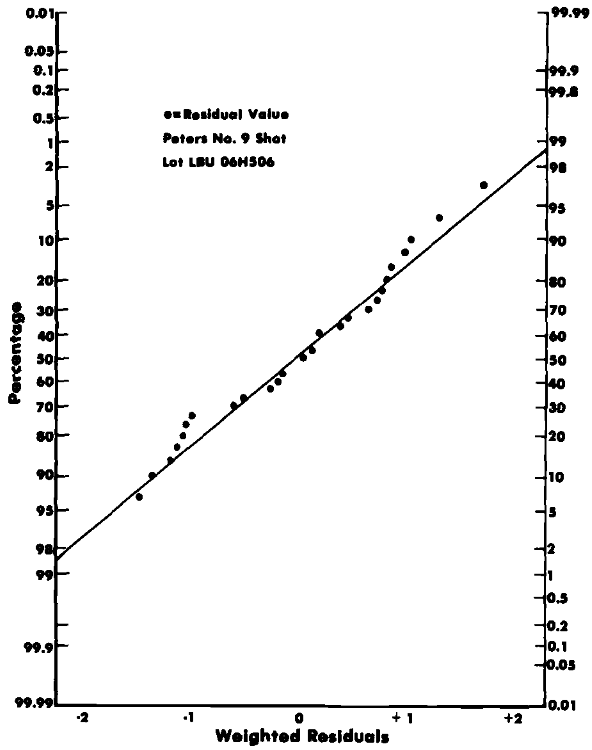

FIG. 9—*Normal plot for Peters No. 9 shot, Lot No. LBU 06H506.*

TABLE 5—*ANOVA for regressions on standard deviations (Federal 00 buckshot cartridges, Lot No. 2127).*

Source of Variation	Degrees of Freedom	Weighted Sum of Squares	Mean Square	F
Regression	1	11.62	11.62	28.85
Residual	3	1.21	0.40	...
Total corrected for mean Y	4	12.83	...	...

Critical value of $F = 10.13$ at the 95% confidence level.

TABLE 6—*ANOVA for weighted regression (Federal 00 buckshot cartridges, Lot No. 2127).*

Source of Variation	Degrees of Freedom	Weighted Sum of Squares	Mean Square	F
Regression	1	331.85	331.85	324.3
Residual	28	28.65	1.02	...
Total corrected for mean Y	29	360.50	...	...

Critical value of $F = 7.64$ at the 99% confidence level.

TABLE 7—*ANOVA for regressions on standard deviations (Peters No. 9 shot cartridges, Lot No. LBU 06H506).*

Source of Variation	Degrees of Freedom	Weighted Sum of Squares	Mean Square	F
Regression	1	3.46	3.46	27.04
Residual	3	0.38	0.12	...
Total corrected for mean Y	4	3.84	...	...

Critical value of $F = 10.13$ at the 95% confidence level.

TABLE 8—*ANOVA for weighted regression (Peters No. 9 shot cartridges, Lot No. LBU 06H506).*

Source of Variation	Degrees of Freedom	Weighted Sum of Squares	Mean Square	F
Regression	1	1015.66	1015.66	969
Residual	28	29.35	1.04	...
Total corrected for mean Y	29	1045.01	...	...

Critical value of $F = 7.64$ at the 99% confidence level.

TABLE 9—*Estimated ranges of fire and confidence intervals for questioned pellet patterns fired with Federal 00 buckshot cartridges, Lot No. 2120.*

Shot	$\sqrt{A}$, cm	Scaled $\sqrt{A}$, cm	Actual Range of Fire, ft[a]	Estimated Range of Fire, ft[a]
1	7.39	8.30	15	15.2 ± 6.2[b]
2	29.83	33.52	48	60.2 ± 30.8
3	6.10	6.86	12	12.7 ± 4.9
4	7.68	8.63	17	15.8 ± 6.5
5	15.07	16.94	31	30.6 ± 14.5
6	9.25	10.40	20	19.0 ± 8.2
7	16.46	18.50	26	33.4 ± 16.0
8	24.80	27.87	46	50.1 ± 25.2
9	9.37	10.53	20	19.2 ± 8.3
10	22.17	24.91	41	44.8 ± 22.3

[a]1 ft = 0.3048 m.
[b]Calculated for a confidence level of 99% ($t = 2.763$).

TABLE 10—*Estimated ranges of fire and confidence intervals for questioned pellet patterns fired with Winchester 00 buckshot cartridges, Lot No. F101UA41.*

Shot	$\sqrt{A}$, cm	Scaled $\sqrt{A}$, cm	Actual Range of Fire, ft[a]	Estimated Range of Fire, ft[a]
1	14.88	20.34	29	36.7 ± 17.8[b]
2	8.93	12.21	20	22.2 ± 9.9
3	7.10	9.70	11	17.7 ± 7.5
4	6.60	9.02	22	16.5 ± 6.8
5	7.24	9.90	14	18.1 ± 7.7
6	10.74	14.68	22	26.6 ± 12.3
7	19.77	27.02	36	48.6 ± 24.4
8	15.78	21.57	35	38.9 ± 19.1
9	9.00	12.30	20	22.3 ± 10.0
10	12.32	16.84	25	30.4 ± 14.4

[a]1 ft = 0.3048 m.
[b]Calculated for a confidence level of 99% ($t = 2.763$).

TABLE 11—*Estimated ranges of fire and confidence intervals for questioned pellet patterns fired with Peters No. 9 shot cartridges, Lot No. LAW 081506.*

Shot	$\sqrt{A}$, cm	Scaled $\sqrt{A}$, cm	Actual Range of Fire, ft[a]	Estimated Range of Fire, ft[a]
1	25.02	22.18	31	27.9 ± 7.0[a]
2	15.68	13.91	20	18.3 ± 5.2
3	12.96	11.50	18	15.5 ± 4.7
4	28.32	25.12	33	31.3 ± 7.6
5	8.27	7.33	13	10.7 ± 3.8
6	12.45	11.04	16	15.0 ± 4.6
7	13.40	11.89	19	16.0 ± 4.7
8	10.41	9.24	14	12.9 ± 4.2
9	21.10	19.60	30	24.9 ± 6.4
10	18.42	16.30	25	21.1 ± 5.7

[a]1 ft = 0.3048 m.
[b]Calculated for a confidence level of 99% ($t = 2.763$).

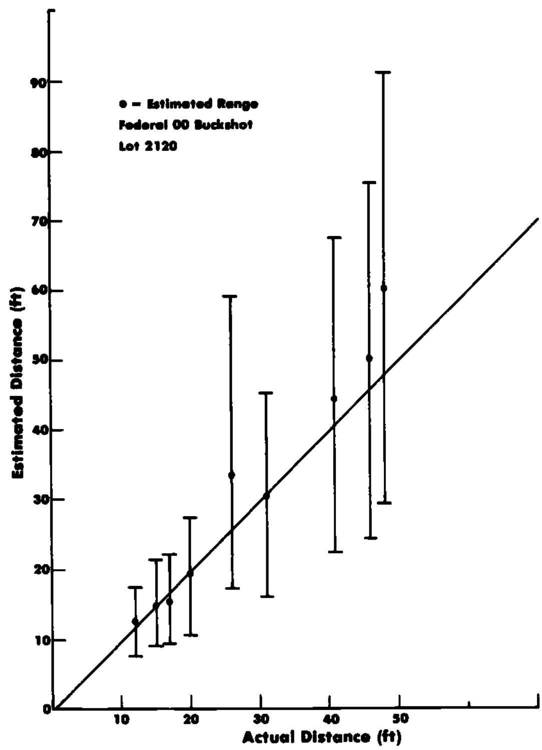

FIG. 10—*Estimated range of fire versus actual range of fire for Federal 00 buckshot, Lot No. 2120. The 99% confidence intervals are shown.*

(50 ft) with ammunition from different lots or brands of ammunition were different at a 95% confidence level. The same results were obtained when the No. 9 birdshot pellet patterns fired with Lot LBU 06H506 were compared with those fired with Lot LAW 081506. Therefore, the ranges of fire for the questioned patterns could not be calculated directly from the results of the regressions carried out on the two 30-shot groups. The assumption was made that the ballistic performance of the shotshells used to fire the questioned patterns and that of the shotshells used to fire the test patterns were sufficiently similar that simple scaling of the spreads of the questioned patterns by a multiplicative factor would permit the use of the regression equations for the test-fired patterns to obtain the estimated ranges of fire.

The ratios of the means of the spreads of the pellet patterns at 15.2 m (50 ft) were used to obtain the scaling factors. The scaled spreads of the questioned pellet patterns were inserted into the appropriate regression equation for the calculation of the estimated ranges of fire. The estimated standard deviations in $\sqrt{A}$ at the estimated ranges of fire were calculated from the regression equations for the standard deviation of $\sqrt{A}$ as a function of range of fire. Finally, the estimated ranges of fire and the associated standard deviations of $\sqrt{A}$ were used to calculate the symmetrical confidence intervals for the estimated ranges of fire at a 99% confidence level. The mathematical details of this procedure have been dealt with elsewhere [*2*]. The results of these calculations are given in Tables 9 through 11.

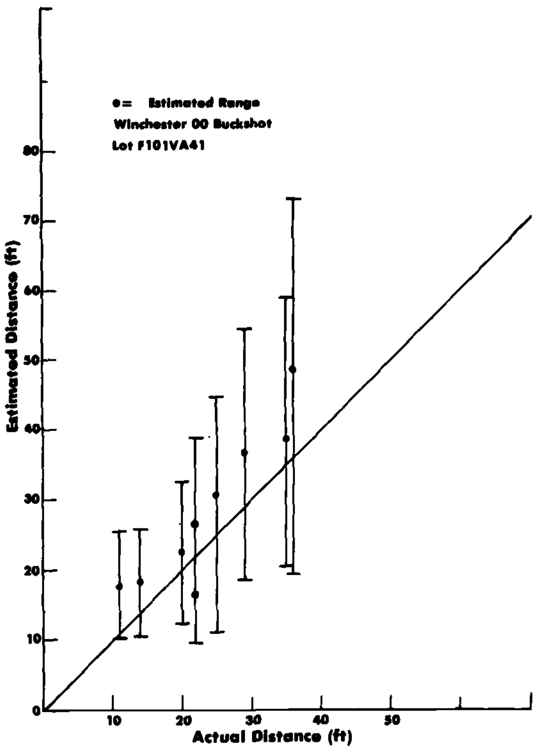

FIG. 11—*Estimated range of fire versus actual range of fire for Winchester 00 buckshot, Lot No. F101UA41. The 99% confidence intervals are shown.*

Results and Discussion

As may be seen in Tables 9 through 11 and in Figs. 10 through 12, in all cases the actual ranges of fire for the questioned pellet patterns fell within the 99% confidence interval of the estimated range of fire. Less variation between the estimated and actual range of fire was observed when the questioned patterns were fired with a different lot of the same type of ammunition produced by the same manufacturer. Greater variation between the estimated and actual ranges of fire was noted when the ammunition was manufactured by a different company.

We observed that the range-of-fire estimates for ranges greater than 9.1 m (30 ft) had very wide confidence intervals. This is consistent with the results of Rowe and Hanson [*3*]. The wide confidence intervals for ranges of fire greater than 9.1 m (30 ft) result from the increased uncertainty in the regression equations at longer ranges and from the increased standard deviation of $\sqrt{A}$.

Conclusion

The results of this study show that usable range-of-fire estimates may be obtained even when only a small number of shotshells of the same lot as that used to fire the questioned

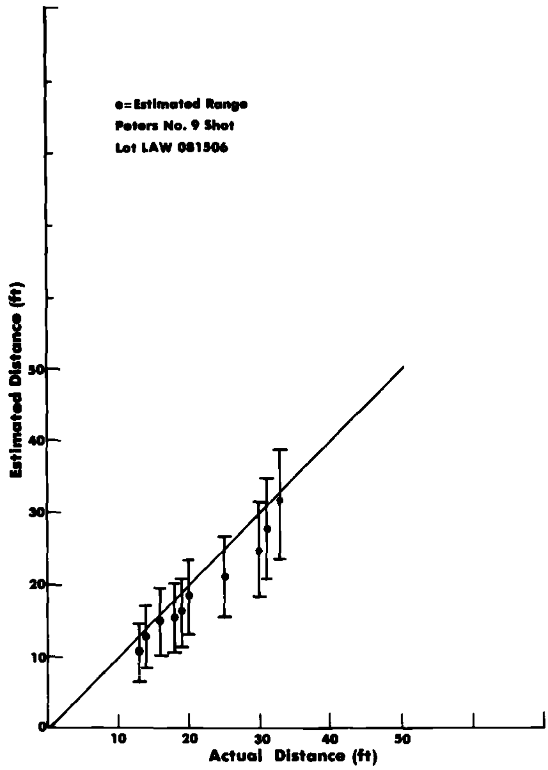

FIG. 12—*Estimated range of fire versus actual range of fire for Peters No. 9 shot, Lot No. LAW 0815506. The 99% confidence intervals are shown.*

pattern are available. The firearms examiner can test fire ammunition of another lot (or even ammunition made by another company) and scale the spreads of the questioned patterns as needed.

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